

Trigonometric Identities

Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \quad \csc x = \frac{1}{\sin x}$$

$$\cos x = \frac{1}{\sec x} \quad \sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{1}{\cot x} \quad \cot x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{1}{\tan x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\sec^2 x - \tan^2 x = 1$$

$$\sec^2 x = \tan^2 x + 1$$

$$\tan^2 x = \sec^2 x - 1$$

$$\csc^2 x - \cot^2 x = 1$$

$$\csc^2 x = \cot^2 x + 1$$

$$\cot^2 x = \csc^2 x - 1$$

Symmetry Identities

$$\sin(-x) = -\sin x \quad \cot(-x) = -\cot x$$

$$\cos(-x) = \cos x \quad \sec(-x) = \sec x$$

$$\tan(-x) = -\tan x \quad \csc(-x) = -\csc x$$

Complement Formulas

$$\sin x = \cos(90-x)$$

$$\cos x = \sin(90-x)$$

$$\tan x = \cot(90-x)$$

$$\cot x = \tan(90-x)$$

$$\sec x = \csc(90-x)$$

$$\csc x = \sec(90-x)$$

Supplement Formulas

$$\sin(\pi-x) = \sin x$$

$$\cos(\pi-x) = -\cos x$$

$$\tan(\pi-x) = -\tan x$$

$$\csc(\pi-x) = \csc x$$

$$\sec(\pi-x) = -\sec x$$

$$\cot(\pi-x) = -\cot x$$

Addition & Subtraction Formulas

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Double-Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 2 \cos^2 x - 1$$

$$\cos 2x = 1 - 2 \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Half-Angle Formulas

$$\sin \frac{1}{2} x = \pm \sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{1}{2} x = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan \frac{1}{2} x = \frac{\sin x}{1 + \cos x}$$

$$\tan \frac{1}{2} x = \frac{1 - \cos x}{\sin x}$$

$$\tan \frac{1}{2} x = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

Product Formulas

$$\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x + y) + \cos(x - y)]$$

$$\sin x \cos y = \frac{1}{2} [\sin(x + y) + \sin(x - y)]$$

Sum Formulas

$$\sin x + \sin y = 2 \sin \frac{1}{2}(x + y) \cos \frac{1}{2}(x - y)$$

$$\sin x - \sin y = 2 \sin \frac{1}{2}(x - y) \cos \frac{1}{2}(x + y)$$

$$\cos x + \cos y = 2 \cos \frac{1}{2}(x + y) \cos \frac{1}{2}(x - y)$$

$$\cos x - \cos y = -2 \sin \frac{1}{2}(x + y) \sin \frac{1}{2}(x - y)$$

Inverse Functions

$$y = \sin^{-1} x \quad \text{if and only if } x = \sin y \\ \text{and } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

$$y = \cos^{-1} x \quad \text{if and only if } x = \cos y \\ \text{and } 0 \leq y \leq \pi$$

$$y = \tan^{-1} x \quad \text{if and only if } x = \tan y \\ \text{and } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$$

SOH-CAH-TOA

$$\sin x = \frac{\textit{opposite}}{\textit{hypotenuse}}$$

$$\cos x = \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

$$\tan x = \frac{\textit{opposite}}{\textit{adjacent}}$$